

## When Contact Ratio Changed

**Sabah M.J.Ali**  
**Professor**  
**Assistant Lecturer**

**Omar D.Mohammad**

**College of Engineering \ University of Mosul**

**[Prosabah7447@yahoo.com](mailto:Prosabah7447@yahoo.com)**  
**[Enggomar@yahoo.com](mailto:Enggomar@yahoo.com)**

### Abstract

The present research work concentrates on the effect of contact ratio change on the stresses generated on meshing gear teeth. Many cases of contact ratio (1.6, 1.7, 1.8, 1.9 & 2.0) have been studied. In each case the value and location of load were determined on the involute profile of meshing tooth. Also the angular position of this tooth for critical loading condition (at the instant when the generated root stresses at the maximum state). (Ansys) programming using F.E.M have been applied for stress analysis on gear model. From this study it is clear that, the highest contact ratio resulted in the lowest generated tooth stresses (i.e. the highest load carrying capacity). This depends on the value, location and direction of load applied on tooth involute.

**Keywords** : Spur gear, Contact ratio, Load sharing, Stress analysis

تقسيم الحمل على أسنان التروس وتحليل الإجهادات عند تغير نسبة التماس

صباح محمد جميل ملا علي

عمر داود محمد

أستاذ

مدرس مساعد

كلية الهندسة / جامعة الموصل

[Prosabah7447@yahoo.com](mailto:Prosabah7447@yahoo.com)

[Enggomar@yahoo.com](mailto:Enggomar@yahoo.com)

### الخلاصة

يتمحور هذا البحث حول دراسة تأثير تغير نسبة التماس على الإجهادات المتولدة في حالة تعشيق تم تحديد قيمة الحمل المسلط وموقعه على سطح السن بالإضافة إلى الوضع الزاوي للسن المعشق لحالة التحميل الحرج (في اللحظة التي تكون فيها أعلى إجهادات متولدة في منطقة). تم تطبيق برمجة (Ansys) التي تعتمد طريقة العناصر المحددة في تحليل الإجهادات. أتضح من الدراسة أن الإجهادات المتولدة في السن المعشق تتناقص عند زيادة نسبة التماس اعتماداً على قيمة الحمل المسلط وموقعه على سطح السن واتجاهه.

Received 3 June 2007

Accepted 31 Dec. 2007

### Symbols

ARC: Arc of contact (mm).

PTH: Path of contact (mm).

C.R: Contact ratio.

$S_1$  : Maximum principal stress  
( $N/mm^2$ )

F: Transmitted load (N)	$S_3$ : Minimum principal stress (N/mm <sup>2</sup> )
$F_n$ : Applied load on gear model(N)	$S_{XY}$ : Maximum shear stress (N/mm <sup>2</sup> )
$F_x$ : X-component of applied load(N)	$S_V$ : Vonmises equivalent stress (N/mm <sup>2</sup> )
$F_y$ : Y-component of applied load(N)	T: Transmitted torque.(N.m)
$P_b$ : Base circular pitch.(mm)	$\omega$ : Angular speed (rad./sec.)
$P_c$ : Circular pitch (mm).	$\phi$ : Pressure angle (degree)

## Introduction

The analysis of stresses generated in gear teeth is considered as a limiting factor for gear designers. Hence according to the analysis of generated stresses, transmitted load, running speed, tooth geometry and other design considerations can be determined. The importance of stress analysis is focused on the determination of stress concentration regions that failure or fracture may initiate at these regions. Fracture of gear tooth can be predicted at tooth root area where tension stresses concentrate; hence cracks may initiate and propagate at root area when maximum generated stress at this area exceeds the yield stress. Moreover, pitting may occurs due to stress concentration at contact area.

L.Wilcox and W.coleman [1] presented a study using F.E.M for the stress analysis of gear tooth and according to the obtained results; they suggested a new formula to determine root stresses. T.Sayama, S.Oda and K.Umezawa [2] presented a study using F.E.M for the analysis of thin rimmed gear stresses; hence the study shows a good approach from the results gotten empirically using strain gauges. A.H.Elkholy [3] introduced a method to determine tooth load sharing especially for high contact ratio spur gearing. S.C.Mohanty [4] suggested an analytical method to calculate the individual tooth load during meshing cycle, also he referred to determination of the locations and sizes of contact zones along the path of contact for high contact ratio gearing ( $3 > C.R > 2$ ).

The previous researches did not study the relation between contact ratio change and generated tooth stresses. In the present research work the contact zones for low contact ratio gearing had been found, assuming that pinion can be engaged with many gears independently with different contact ratio. Many cases of the contact ratio (1.6, 1.7, 1.8, 1.9 & 2.0) had been studied, as these ratios are the most used in gearsets. In each case the tooth stresses had been calculated to find the effect of the contact ratio of gearing on generated stresses.

## Contact Ratio

The power to weight ratio of gear trains has increased steadily over the years. One of the recent improvements lies in the area of design, where sharing the load among more teeth (with increasing the contact ratio), it is possible, under certain conditions, to increase the load carrying capacity of a gear set without substantially increasing its weight. Contact ratio can be defined as a number of teeth in contact as these teeth pass through the contact zone. It is impractical to make C.R less than unity. For low contact ratio gearing the number of meshing teeth alternates between one and two.

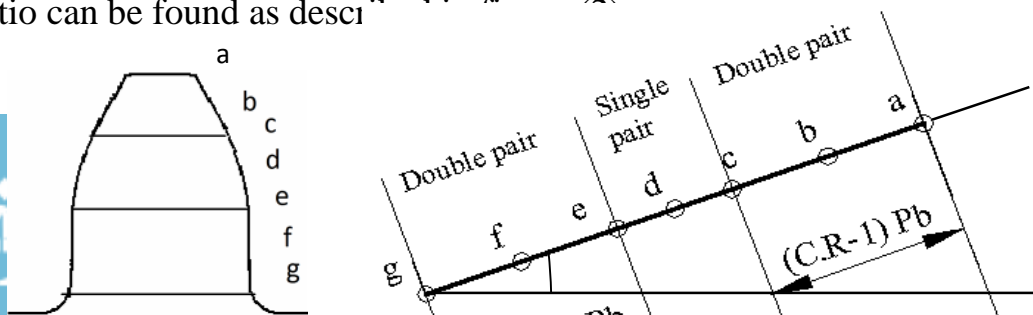
A contact ratio of (1.0) means that only one pair is engaged at all times during the course of action. This is undesirable because slight errors in the tooth spacing will cause oscillations in the velocity, vibration and noise. For satisfactory performance of power transmitting gears a value of (1.4) is used as a practical minimum. A lower contact ratio also necessitates a higher degree of accuracy in meshing to ensure quiet running of the gear set. A contact ratio of (2.0) means that there are always two pairs in contact, i.e. at the instant when one pair goes out of contact, a new pair comes into contact [5].

In low contact ratio gearing, the load is transmitted by single pair of teeth for part of the period of engagement, and by two pair of teeth during rest of the period. Where as in the beginning of contact there are two pair in contact continuing in meshing for a specified period then one of them goes out of contact, and the other pair continues in meshing alone, until a new pair comes into action [6]. For example a contact ratio of (1.6)

means that in the beginning of the path of contact, there are two pairs continuing in meshing 60% of the base circular pitch along the path of contact. Then one pair goes out of contact and the other pair continues in meshing alone for the rest 40 % of the base circular pitch, after this a new other pair comes into contact. By this method the engagement will be repeated, as shown in figure (1).

$$C.R = \frac{ARC}{P_c} = \frac{PTH}{P_c \cdot \cos(\phi)} = \frac{PTH}{P_b} \dots\dots\dots(1) [7]$$

In general the contact zones along the line of action for low contact ratio can be found as described in figure (2).



## Load Sharing

From figure (2) it is clear that the first point of action “g” which is located on the first meshing tooth is associated with point “c” which is located on the second meshing tooth of the same pinion. Therefore the load will be shared between these two points, similarly points “f” and “b”, also “e” and “a”. Then point “a” will go out of contact, therefore the full load will be applied on point “e”, also point “d” until the contact being at point “c”, then a new meshing tooth comes in to contact.

In low contact ratio gearing, and when a single pair of teeth is engaged, this pair transmits the full load or the full load is then applied on the one meshing tooth only. Almost, critical conditions (for maximum generated root stresses) occur in the one pair contact zone. When double pair of teeth are engaged, the transmitted load will be divided between two meshing teeth. Practically the load is not divided fairly; load sharing depends on contact ratio value and stiffness of meshing tooth at point of

application of load [3]. In figure (3) the load sharing is drawn against the path of contact for low contact ratio sharing.

## Gear Model

This study using F.E.M had been applied on a pinion spur gear model running at (5000 r.p.m) transmitting 400 kW. All data relating to pinion gear is shown in table (1).

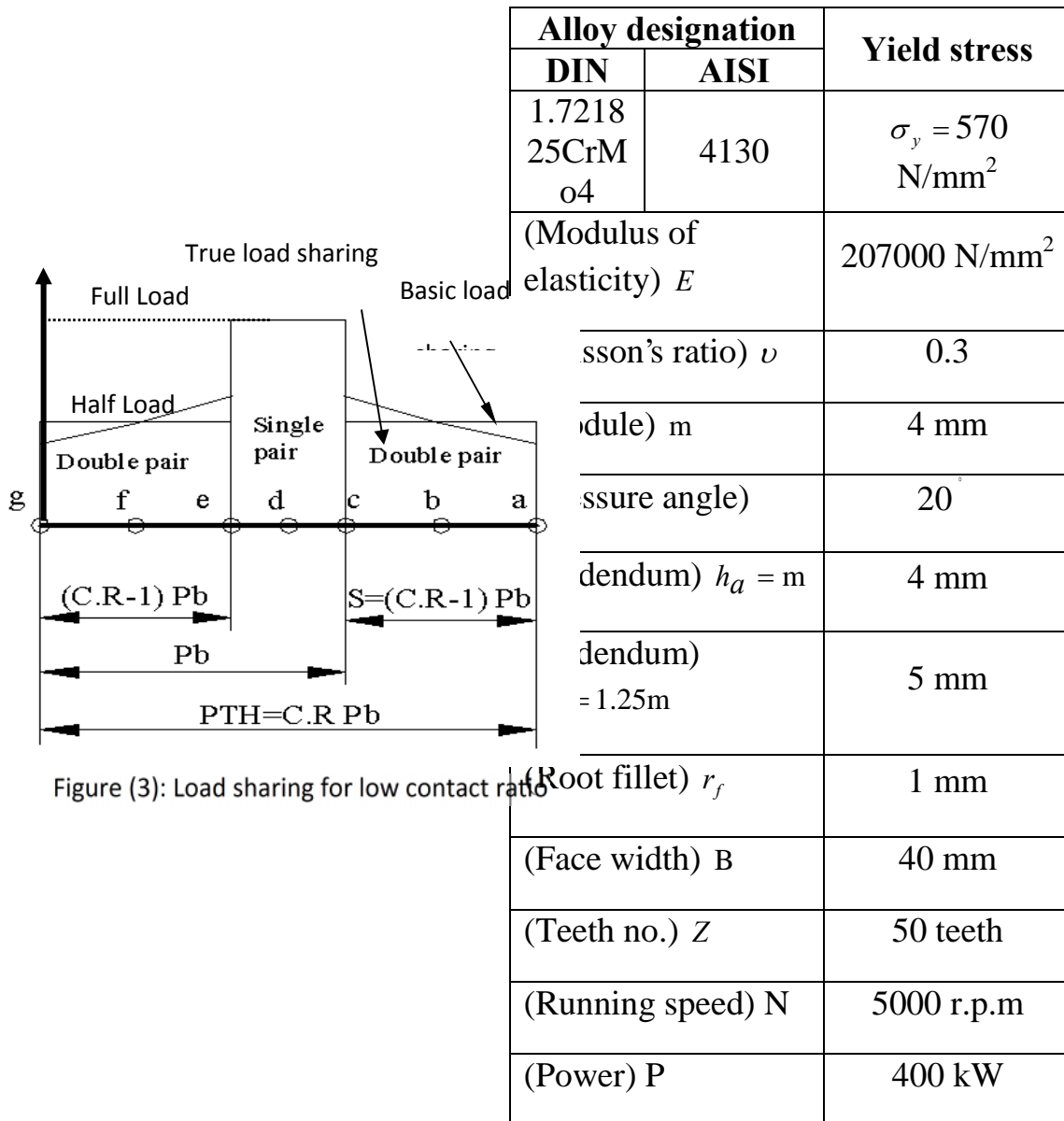


Figure (3): Load sharing for low contact ratio

Table (1): Data

relating to pinion gear

### Transmitted Load

Gears are used to transmit mechanical power and this requires applying mechanical torque that can be calculated as following, [6]:

$$T = \frac{P}{\omega} \dots\dots\dots(2)$$

Then the normal load applied on meshing teeth can be found as following, [6]:

$$F = \frac{T}{r_b} \dots\dots\dots(3)$$

Where:  $r_b = r \cdot \cos(\phi) \dots\dots\dots(4)$

The stress analysis problem in this study is assumed as a plane elastic problem, since the applied transmitted load is assumed to be distributed uniformly across the width of the meshing tooth. Therefore the load had been depended per unit width of tooth as following:

$$F_n = \frac{F}{B} \dots\dots\dots(5)$$

Thus the component ( $F_x$ ) and ( $F_y$ ) will be

$$F_x = F_n \cdot \cos(\phi) \dots\dots\dots(6)$$

$$F_y = F_n \cdot \sin(\phi) \dots\dots\dots(7)$$

When the contact ratio being (2.0), then the applied load is half of the transmitted load. Hence ( $F$ ) will be half of its value in (5), then each of ( $F_x$ ) and ( $F_y$ ) will be also half of its value.

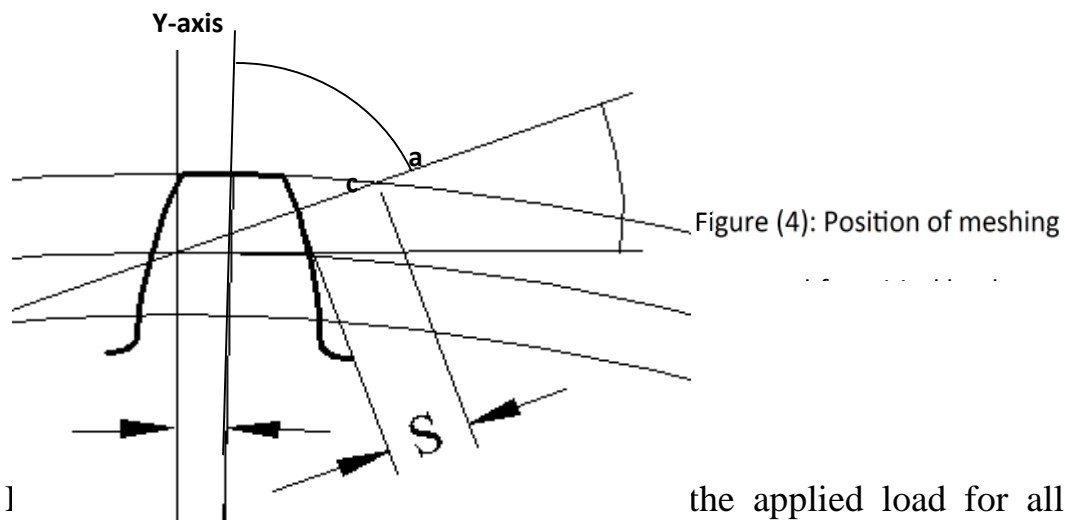
When the contact ratio changes, the path of contact and the load sharing will be changed too, as explained earlier. To explain how to determine the location of the critical load and the angular position of the meshing tooth for each case of contact ratio, see figure (3). It is clear that the meshing at point “c” with full applied load on meshing tooth, leads to a maximum generated stresses in root area, hence the load applied on



point “c” is the *critical* load. Meshing at point “c” creates the largest possible bending moment with single tooth contact; hence point “c” is called the highest point of single tooth contact. To determine location of point “c” on the path of contact, (S) must be calculated, and then the angular position of meshing tooth can be measured by angle ( ), as shown in figure (4).

Table (2): The data required for determining the critical conditions.

Case no.	C.R	S (mm)	(x , y) of point “c”	(degree)
1	1.6	7.08	-2.94 , 101.13	-0.23
2	1.7	8.26	-1.83 , 100.73	0.49
3	1.8	9.44	-0.73 , 100.32	1.21
4	1.9	10.62	0.38 , 99.92	1.93
5	2.0	0.00	-9.6 , 103.56	-4.59



the applied load for all contact points is half of the transmitted load, then the critical load will be at the tip of the meshing tooth (at point “a”). Table (2) shows all data required for determining the critical conditions of loading for all studied cases of contact ratio.

### Ansys Programming

Ansys package is one of the efficient engineering programming, which has the ability to solve many engineering problems using F.E.M.

This programming has easiness in use, flexibility in application and reliability for solving problems in engineering fields such as stress analysis field. The tooth model in this study had been assumed as a plane stress problem due to small width of tooth compared with radius of gear. As shown in figure (5-a), the model is meshed with 2.D triangular 6-node elements. For contact ratios (1.6, 1.7, 1.8 & 1.9) the critical load occurs with single meshing pair, therefore the model in this case will be depended with one tooth as shown in figure (5-b). Otherwise, for contact ratio (2.0) the critical load occurs with two meshing pair, therefore the model in this case will be depended with two teeth, as shown in figure (5-c). All nodes on the three sides (A, B, C) of the model indicated in figure (5) are kept fixed as boundary conditions for solution. For each case of contact ratio, the load is applied at locations (x , y) indicated in table (2), when tooth angular position make ( ).

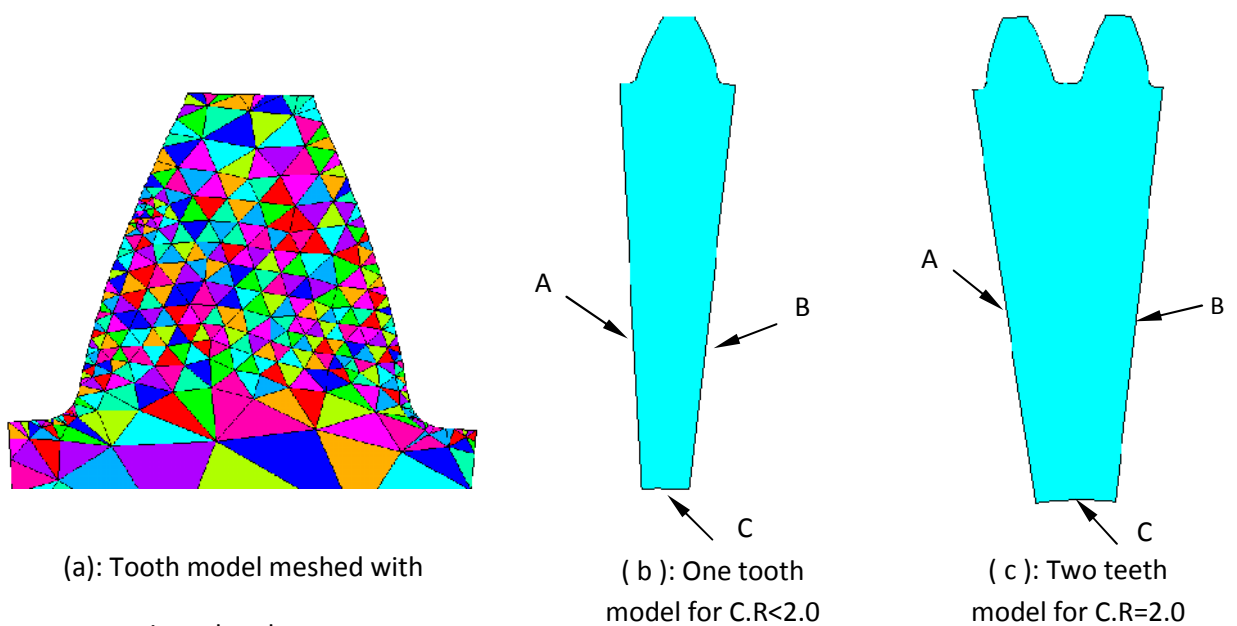


Figure (5): gear tooth model



## Results and Discussions

Curves plots can be used for observing changes of stresses generated in tooth model with change of the contact ratio.

**Contact area:** Figures (6-a), (6-b) and (6-c) show the change of ( $S_3$ ), ( $S_{XY}$ ) and ( $S_V$ ) respectively against contact ratio change for the contact area. From these plots, it is clear that the maximum generated stresses in this area decrease with increasing the contact ratio, this due to increase of direction of applied load relative to the axis of the meshing tooth (increase of “ ” indicated in figure (4)).

**Root Area:** Figures (7-a), (7-b) and (7-c) show the change of ( $S_1$ ), ( $S_{XY}$ ) and ( $S_V$ ) respectively against contact ratio change for the root area. From these plots, it is clear that the maximum generated stresses in this area decrease with increasing the contact ratio, this due to decrease of

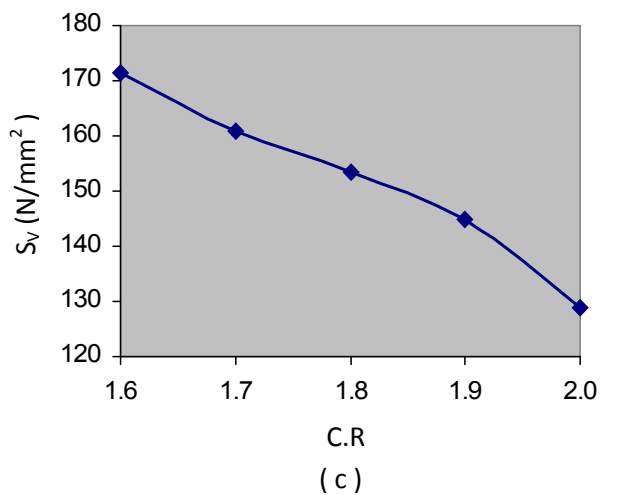
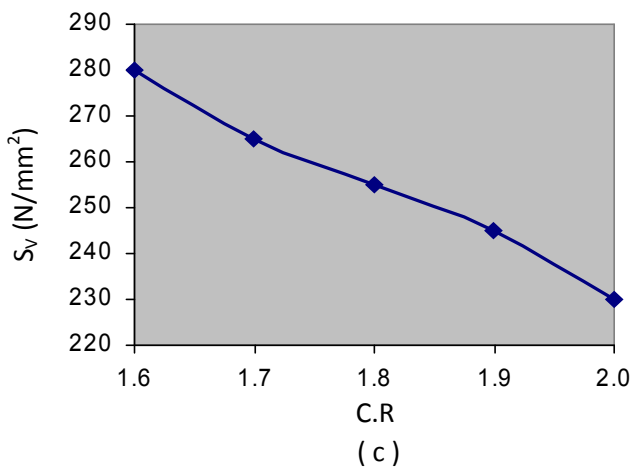
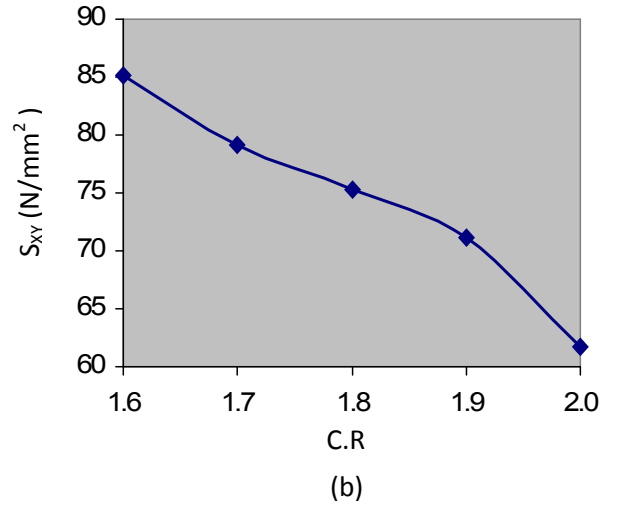
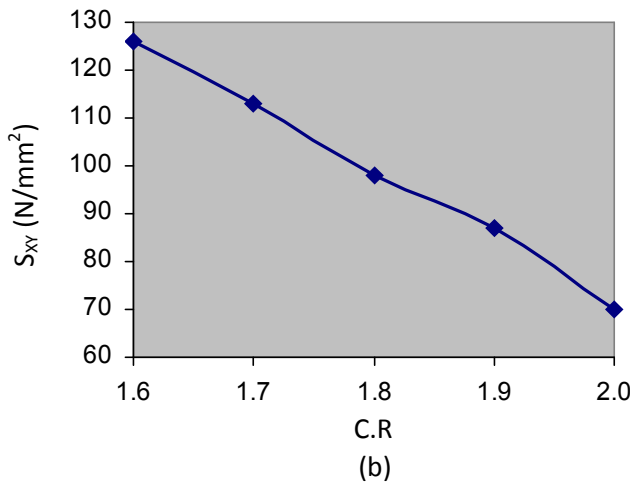
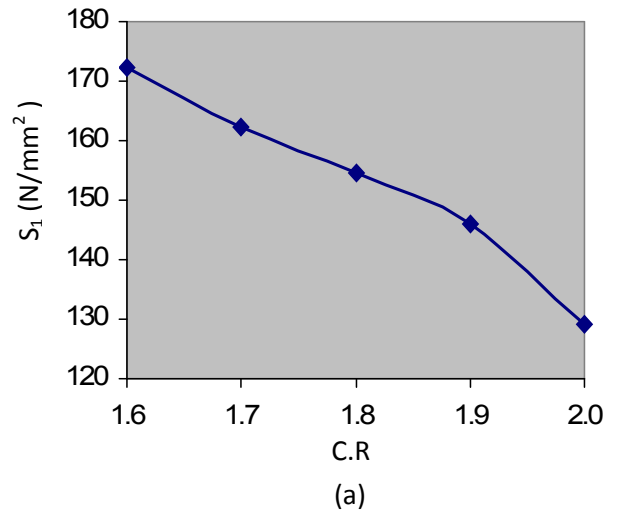
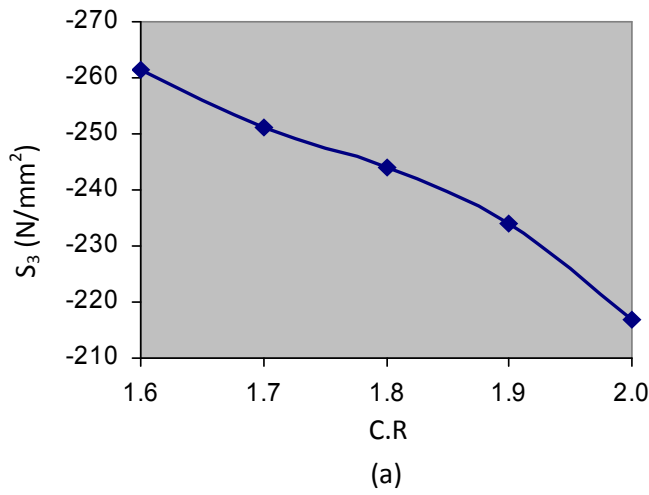


Figure (6): Change of stresses at contact area with contact ratio

Figure (7): Change of stresses at root area

force moment (decrease the distance measured between the point of application of critical load and the root of meshing tooth).

Each one of the three previewed stresses is considered as a criterion of one of the theories of failure. However the three stresses increased with decreasing the contact ratio, then failure is expected for lower contact ratio.

### Conclusions

1. The stresses generated on spur gear teeth change with changing the contact ratio of gearing, where as the maximum generated stresses observed in contact and root areas decrease with increasing the contact ratio.
2. When the contact ratio changed from (1.9) to (2.0), the decrease of stresses was more than the decrease of stresses when the contact ratio changed between any two other successive cases. This because that the critical load is half of the transmitted load when (C.R=2.0).
3. Failure of gear teeth is expected for lower contact ratio.

### References

1. Wilcox L., and Coleman W., "Application of Finite Element to the Analysis of Gear Tooth Stresses ", Journal of Engineering for Industry, Vol. 95, No. 4, pp. 1139 -1148, 1973.
2. Sayama T., Oda S., and Umeezawa K., " Root Stresses and Bending Fatigue Strength of Welded Structure Gears ", International Symposium on Gearing & Power Transmissions, Japan, Tokyo, 1981.
3. Elkholy A.H., " Tooth Load Sharing in High Contact Ratio Spur Gears", Journal of Mechanisms, Transmissions, and Automation in Design", Vol. 107, No. 1, pp. 11-16, 1985.
4. Mohanty S.C., "Tooth Load Sharing and Contact Stress Analysis of High Contact Ratio Spur Gears in Mesh", national Convention of Mechanical Engineers, Roukela, 2002.
5. Norton R.L., "Design of Machinery", McGraw-Hill Companies, Inc, U.S.A., NewYork, 2001.

6. Maitra G.M., “ Handbook of Gear Design ”, Tata McGraw-Hill Company, India, NewDelhi, 1997.
7. Hannah J., and Stephens R.C., “Mechanics of Machines”, Edward Arnold, Ltd., U.K., London, 1975.

The work was carried out at the college of Engg. University of Mosul